On the Near-field of an Antenna and the Development of New Devices

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Abstract

Dipole antennas have been invented a hundred years ago. Nevertheless, what really happens in their proximity during emission (the so-called near-field) is still an open question and the many explanations put forth are not fully convincing. Subject to specific conditions the signal present on the conductor assumes the properties of a pure electromagnetic wave. We would like to give our point of view on the modality of this transition, with the hope of providing suggestions for ameliorating or projecting new devices.

Keywords: dipole antennas, near-field, non solenoidal fields.

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1 Non divergenceless fields

We start by introducing the model equations proposed in [3]. They include Maxwell's equation as a particular case allowing an impressive enlargement of the space of solutions. The main important consequence of this approach is the possibility of handling photons (i.e., non-diffusive electromagnetic emissions having compact support) in the classical fashion, reopening the old diatribe on the wave-particle duality. This is not however the subject that we are discussing here. We want instead to analyze the impact of this alternative formulation on the study of simple antennas.

Let us first remark that the revised model shares with Maxwell's all the good properties and characterizations, presenting no weak points in this comparison. One of the crucial steps is to assume that, even in vacuum, the electric field may have divergence different from zero. The reasons for this choice are thoroughly documented in [3] and successive papers (see [4], [5], [6]). We would like to stress

here that, also in the framework of simple applications, this assumption is not just a theoretical exercise, but a real necessity. We invite then the reader to accept possible explanations regarding the functioning mechanism of antennas, in the light of a solid theory where non divergenceless (i.e.: non solenoidal) electric fields are allowed.

Let us begin by writing down the equations in the case of *free-waves*, which form a subset of all possible electromagnetic manifestations. Denoting by $\mathbf{E} = (E_1, E_2, E_3)$ the electric field and by $\mathbf{B} = (B_1, B_2, B_3)$ the magnetic field, we have:

$$\frac{\partial \mathbf{E}}{\partial t} \;=\; c^2 \mathrm{curl}\mathbf{B} \;-\; \rho \mathbf{V} \tag{1.1}$$

$$\frac{\partial \mathbf{B}}{\partial t} \;=\; -\,\mathrm{curl}\mathbf{E} \tag{1.2}$$

$$\mathrm{div}\mathbf{B} \;=\; 0 \tag{1.3}$$

$$\rho\,(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \;=\; 0 \tag{1.4}$$

where we set: $\rho = \mathrm{div}\mathbf{E}$ (this is a definition, not a ruling equation). Moreover, c denotes the speed of light and $\mathbf{V} = (V_1, V_2, V_3)$ is a velocity field in such a way that the triplet $(\mathbf{E}, \mathbf{B}, \mathbf{V})$ is right-handed. The magnitude of $\mathbf{V}$ is equal to c. Equation (1.1) turns out to be the Ampère law for a free flowing immaterial current with density ρ. It is clear that by imposing $\rho = 0$ (this now becomes an equation) we reobtain the classical Maxwell's system. Equations (1.2) and (1.3) do not need explanation. Equation (1.4) characterizes free-waves and, as we will see later, can be assimilated to a sort of geometrical constraint. More explanations regarding its physical meaning can be found in [3] and [4].

We do not waste time here for a discussion of the physical and theoretical properties of the above set of equations. We just mention a few meaningful facts. A continuity equation for ρ is already contained in equation (1.1). In fact, by taking its divergence, one obtains:

$$\frac{\partial \rho}{\partial t} \;=\; -\,\mathrm{div}(\rho \mathbf{V}) \;=\; -\,\rho\,\mathrm{div}\mathbf{V} \;-\; \nabla\rho \cdot \mathbf{V} \tag{1.5}$$

By scalar multiplication of (1.4) by $\mathbf{V}$ and $\mathbf{B}$, we get respectively:

$$\mathbf{E} \cdot \mathbf{V} \;=\; 0 \qquad\qquad \mathbf{E} \cdot \mathbf{B} \;=\; 0 \tag{1.6}$$

Afterwards, with the same calculations relative to the Maxwell's case, the following energy relation holds:

$$\frac{1}{2}\frac{\partial}{\partial t}(|\mathbf{E}|^2 + c^2|\mathbf{B}|^2) + c^2\mathrm{div}(\mathbf{E} \times \mathbf{B}) \;=\; 0 \tag{1.7}$$

where $\mathbf{E} \times \mathbf{B}$ is the Poynting vector.

In the system of time-space coordinates where $x_0 = ct$, we can consider the electromagnetic stress tensor $T^{\alpha\beta}$. It is known that Maxwell's equations are

compatible with the conservation law: $\partial_\alpha T^{\alpha\beta} = 0$. As a matter of fact, by summing up on the index β, one has:

$$\frac{\partial T^{0\beta}}{\partial x_\beta} = \frac{1}{2c}\frac{\partial}{\partial t}(|\mathbf{E}|^2 + c^2|\mathbf{B}|^2) + c \ \mathrm{div}(\mathbf{E}\times\mathbf{B}) \tag{1.8}$$

$$\left(\frac{\partial T^{1\beta}}{\partial x_\beta}, \frac{\partial T^{2\beta}}{\partial x_\beta}, \frac{\partial T^{2\beta}}{\partial x_\beta}\right) = \left(\frac{\partial \mathbf{B}}{\partial t} + \mathrm{curl}\mathbf{E}\right)\times\mathbf{E}$$

$$-\left(\frac{\partial \mathbf{B}}{\partial t} - c^2\mathrm{curl}\mathbf{B}\right)\times\mathbf{B} \ + \ \mathbf{E} \ \mathrm{div}\mathbf{E} \ + \ c^2\mathbf{B} \ \mathrm{div}\mathbf{B} \tag{1.9}$$

Therefore, if Maxwell's equations in vacuum are satisfied all the above expressions are zero (recall (1.7)). On the other hand, by adding and subtracting the term $(\mathrm{div}\mathbf{E})\mathbf{V}$, (1.9) takes the form:

$$\left(\frac{\partial T^{1\beta}}{\partial x_\beta}, \frac{\partial T^{2\beta}}{\partial x_\beta}, \frac{\partial T^{2\beta}}{\partial x_\beta}\right) = \left(\frac{\partial \mathbf{B}}{\partial t} + \mathrm{curl}\mathbf{E}\right)\times\mathbf{E}$$

$$-\left(\frac{\partial \mathbf{B}}{\partial t} - c^2\mathrm{curl}\mathbf{B} + (\mathrm{div}\mathbf{E})\mathbf{V}\right)\times\mathbf{B} + (\mathbf{E}+\mathbf{V}\times\mathbf{B})\mathrm{div}\mathbf{E} \ + \ c^2\mathbf{B} \ \mathrm{div}\mathbf{B} \tag{1.10}$$

which is now compatible with the new set of equations, so justifying their origin.

All these considerations can be extended to general metric spaces by writing the equations in covariant form. Lorentz invariance and the existence of a Lagrangian have been also considered, though these issues are out of the scope of this paper.

Finally, let us note that a vector wave equation for $\mathbf{E}$ is not available. We claim that this is not a trouble but a key point in the success of the new approach. Solutions of the wave equation tend to diffuse all around (without energy dissipation anyway) and this is one of the reasons for the impossibility to include compact-support solitary waves in a classical theory. According to the new set of equations, we can differentiate (1.1) with respect to time. Using (1.2) one gets:

$$\begin{aligned}\frac{\partial^2\mathbf{E}}{\partial t^2} &= \ -c^2\mathrm{curl}(\mathrm{curl}\mathbf{E}) \ - \ \frac{\partial(\rho\mathbf{V})}{\partial t} \\ &= \ c^2\Delta\mathbf{E} \ - \ c^2\nabla\rho \ - \ \frac{\partial\rho}{\partial t}\mathbf{V} - \ \rho\frac{\partial\mathbf{V}}{\partial t}\end{aligned} \tag{1.11}$$

From the continuity equation (1.5), one obtains:

$$\frac{\partial^2\mathbf{E}}{\partial t^2} \ = \ c^2\Delta\mathbf{E} \ + \ [(\nabla\rho\cdot\mathbf{V})\mathbf{V} \ - \ c^2\nabla\rho] \ + \ \rho\mathbf{V} \ \mathrm{div}\mathbf{V} - \ \rho\frac{\partial\mathbf{V}}{\partial t} \tag{1.12}$$

yielding the usual wave equation for $\rho = 0$. In order to understand the importance of (1.12) let us consider a simple example in cartesian coordinates. Suppose that $\mathbf{E} = (u, 0, 0)$, for a certain u not depending on y. Suppose that

$\mathbf{V} = (0, 0, c)$ (note that $\mathbf{E}$ is orthogonal to $\mathbf{V}$ in accordance to (1.6)). We have: div$\mathbf{V} = 0$, $\partial\mathbf{V}/\partial t = 0$ and $\rho = \partial u/\partial x$. By substituting into (1.12) we get:

$$\frac{\partial^2 u}{\partial t^2} \ = \ c^2 \ \frac{\partial^2 u}{\partial z^2} \tag{1.13}$$

This is a scalar wave equation where the solution is shifting along the z-axis, which is the one individuated by the vector $\mathbf{V}$. This is different from the canonical wave equation, that should also include a second derivative with respect to the variable x. The term $[(\nabla\rho \cdot \mathbf{V})\mathbf{V} - c^2\nabla\rho]$ on the right-hand side of (1.12) is exactly placed with the purpose of cancelling from the Laplacian the unwanted derivatives. A similar result for ρ is obtained by plugging $\mathbf{V} = (0, 0, c)$ in (1.5), which brings to: $\partial\rho/\partial t = -c\partial\rho/\partial z$, where the derivative with respect to x does not appear.

Hence, as far as the variable x is concerned, we have no restrictions on u. Let us also observe that the only solution compatible with $\rho = 0$, i.e. belonging to the solution space of Maxwell's equations in vacuum, imposes that u must be constant in the variable x. We get in this way an unbounded plane wave, showing how little are the degrees of freedom when working with fields with zero divergence.

2 The classical dipole antenna

Let us now face the discussion of the classical dipole antenna. The equations introduced in the previous section teach us how to get perfect spherical wave-fronts with no approximation. These are good for the description of the travelling wave in the far-field.

As customary, computations are carried out in spherical coordinates (r, θ, ϕ). We define the fields:

$$\mathbf{E} \ = \ \Big(0, \ \frac{cf(\theta)}{r} g(t - r/c), \ 0\Big) \qquad \mathbf{B} \ = \ \Big(0, \ 0, \ - \frac{f(\theta)}{r} g(t - r/c)\Big) \tag{2.14}$$

where f and g are arbitrary functions. The profile g controls the time-dependent amplitude of the wave and f its energy distribution on each spherical wave-front. In addition, we set $\mathbf{V} = (c, 0, 0)$ (the transport velocity is along the radial direction). By direct substitution in the model equations, one checks that this setting is an admissible solution.

We have that $(\mathbf{E}, \mathbf{B}, \mathbf{V})$ is an orthogonal triplet with:

$$|\mathbf{E}| \ = \ |c\mathbf{B}| \tag{2.15}$$

compatibly with the impedance of free space of about 376Ω.

With the setting in (2.14), the only way to get a solution satisfying $\rho = 0$ is to impose $f(\theta) = 1/\sin\theta$, causing a singularity along the vertical axis. Of course, if we drop the condition $\rho = 0$, we enjoy much more freedom. In

particular, one can set $f(\theta) = \sin\theta$, which is the standard far-field solution of a half wave-length dipole. One could argue that the last solution is obtained from the Maxwell's model by eliminating negligible terms (take the Hertz solution for an infinitesimal dipole and get rid of the components decaying faster than $1/r$ for $r \to +\infty$). From our point of view, why insisting on approximated solutions when the new model accepts them as exact solutions?

The real problem is to understand what happens in the near-field and, evidently, our dipole cannot be infinitesimal. The emitting device is going to be the segment $[-d/2, d/2]$ in the direction of the vertical axis. For simplicity we assume that $2d$ is the wave-length of an emitted monochromatic frequency $\omega/2\pi$. In other words, we suppose that $g(t - r/c) = \sin\omega(t - r/c)$ and $d = \pi c/\omega$.

The first problem is to discuss boundary conditions. The antenna can be seen as an oscillating chord, with standing nodes at the endpoints. From the solution of a one-dimensional wave equation, one recovers that the time-dependent datum to be assigned on the vertical segment is: $\sin(\omega t)\cos(\omega r/c)$, for $r \le d/2$. This behavior is in general referred to the current intensity circulating within the wire.

We would like to follow a different path. To this end we take into consideration the model equations. As observed at the end of the previous section, the electric field comes from the resolution of a wave equation where on the right-hand side there is not a full Laplacian in spherical coordinates but only second derivatives in the direction of motion. We note that $\mathbf{E}$ does not depend on ϕ and that the only component different from zero is the one relative to the angle θ. Thus, $\mathbf{E}$ is of the form $(0, u, 0)$, where u restricted to the antenna satisfies a "truncated" wave equation of the following type:

$$\frac{\partial^2 u}{\partial t^2} \;=\; \frac{c^2}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) \tag{2.16}$$

The solution of the above equation is related to spherical Bessel functions. In explicit terms, we find out that:

$$u \;=\; \sin(\omega t)\frac{\cos(\omega r/c)}{r} \qquad \text{for} \;\; r \le d/2 \tag{2.17}$$

This is true up to a multiplicative constant which has to be regulated according to the power of the feeding source. The proposed behavior is slightly different from the usual one since there is an extra r at the denominator, producing a faster decay as the electric signal moves towards the extremes of the antenna. On the other hand, such an alternative proposal takes into account that the device is not separated from the general context, being embedded in a spherical environment. In (2.17), we have a singularity for r tending to zero; we know however that there is a small gap between the two arms of the antenna, so that one can safely stay away from the critical region.

If u is the boundary datum, we may ask ourselves what is going to be the behavior of the fields as we leave the antenna. We make the following guess:

$$\mathbf{E} \;=\; \left(0,\; c\frac{\sin(\omega t)\cos(\omega r/c)}{r\sin\theta},\; 0\right) \qquad \mathbf{B} \;=\; \left(0,\; 0,\; -\frac{\cos(\omega t)\sin(\omega r/c)}{r\sin\theta}\right) \tag{2.18}$$

This is in perfect agreement with Maxwell's equations (i.e.: $\rho = 0$). To avoid singularities, we have to assume that $r \sin\theta$ has to be bigger that a quantity ϵ, which is linked to the width of the antenna wire. Moreover, we shall require that $r \leq d/2$. This means that we are staying in a sort of spherical resonant cavity. The electric and magnetic fields have a phase difference of 90 degrees, therefore this cannot be a travelling wave (note instead that the fields in (2.14) have no phase lag). This oscillating situation is associated with the boundary constraints and the corresponding fields do not go anywhere. Indeed the boundary datum does not "shift", representing on the contrary a standing wave.

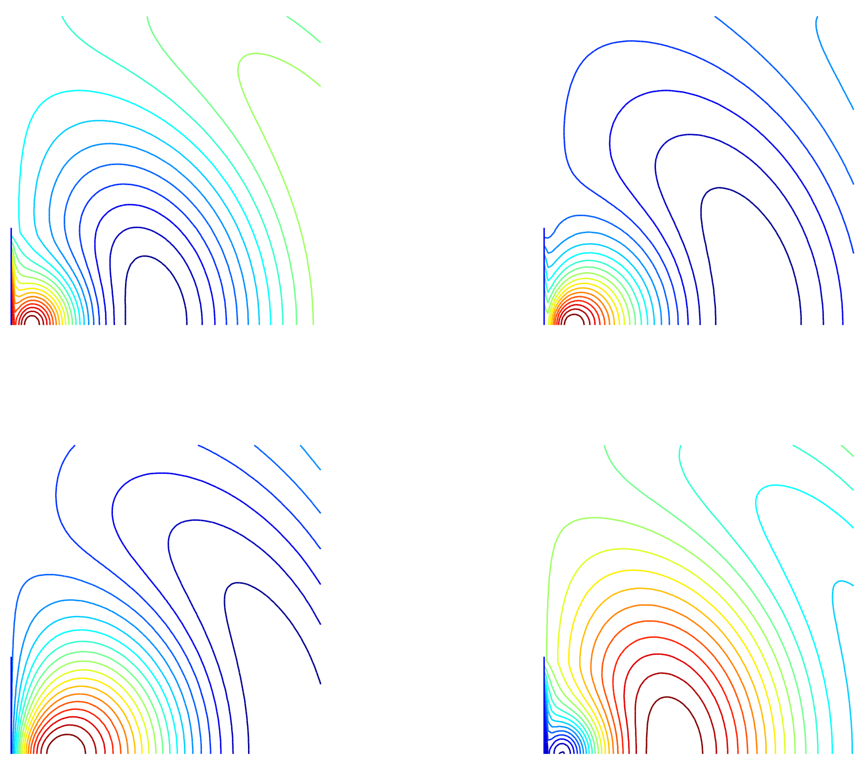

Figure 1: Magnitude of E_2 in (2.19) at different times, for a suitable choice of the constants k_1 and k_2. The upper arm of the dipole is visible on the bottom-left. There is a kind of fictitious bubble separating the guided region from the one where the wave is free. The reader should not confuse the above lines, which are the level sets of the scalar function $|\mathbf{E}|$, with the lines of force of the field, which are distributed on perfect spherical surfaces ($\mathbf{E}$ only has the θ component) as seen in figure 2.

We can pose at this point the crucial question. Why and how the fields circulating in the neighborhood of the antenna start becoming a free electromagnetic emission? The answer we propose is based on a linear combination of the fields in (2.14) and (2.18). Let us observe that the operation of summing up a solution having $\rho = 0$ and a solution having $\rho \neq 0$ is compatible with the model equations, also if these are of nonlinear type. Explicitly, for the second component of the electric field we have:

$$E_2 \;=\; k_1 \frac{f(\theta)}{r} \sin\omega(t - r/c) \;+\; k_2 \frac{\sin(\omega t)\cos(\omega r/c)}{r \sin\theta} \tag{2.19}$$

where k_1 and k_2 are arbitrary constants to be set up based on the power transferred to the antenna device and its geometrical properties (width of the wires and size of the central gap). For the half wave-length dipole we suggest to take $f(\theta) = \sin\theta$. The evolution of this kind of wave can be seen in figure 1. Figure 2 shows the orientation of $\mathbf{E}$.

To recap, the first part in (2.19) is the effective travelling wave and continues to survive for $r > d/2$. Its boundary conditions are zero and remain zero also when there is no more boundary, in order to guarantee the continuity of the spherical wave at the poles. To be more precise, the function f in (2.19) should vanish for ϵ and $\pi - \epsilon$, where 2ϵ is the diameter of the conducting wire. The second part is active only for $r < d/2$ and is compatible with the boundary conditions. It is the standard field produced by an oscillating current in a wire under resonant conditions and follows Maxwell's equations. The global solution is continuous but not smooth for $r = d/2$. The real wave probably follows a better behavior, which also depends on the degree of smoothness of the wires at the endpoints. This is what we can do using simple analytic functions; the only way to get more reliable information is to use numerical computations.

Let us add more explanations. For reasons that will be clarified later, in the narrow gap of the feeding area electric and magnetic fields are generated. Part of this message displays no phase lag, becoming the actual signal that in the end propagates as a free-wave. The rest of the effective energy will be dissipated along the arms. Understanding what happens in this gap is crucial for the comprehension of the entire mechanism of emission. We return on this subject in section 4.

From the production site to the open space, there is the need of a transition zone (the near-field) before the wave can effectively be expelled. As a matter of fact, the guided-wave flowing in the feed line (usually a coaxial wire) satisfies Maxwell's equations ($\rho = 0$). This is possible because one can define the internal boundary conditions accordingly. But, when the wave is completely free there is no boundary and we need to assume that $\rho \neq 0$, entering the regime of the modified model equations. We reinforce this statement by recalling that one cannot comb a sphere (the magnetic field follows the parallels maintaining zero divergence, the electric field is tangent to the meridians and is obliged to be non divergenceless). We need then an appropriate zone to realize smoothly the

passage from $\rho = 0$ to $\rho \neq 0$, and spread the non divergenceless property along spherical surfaces.

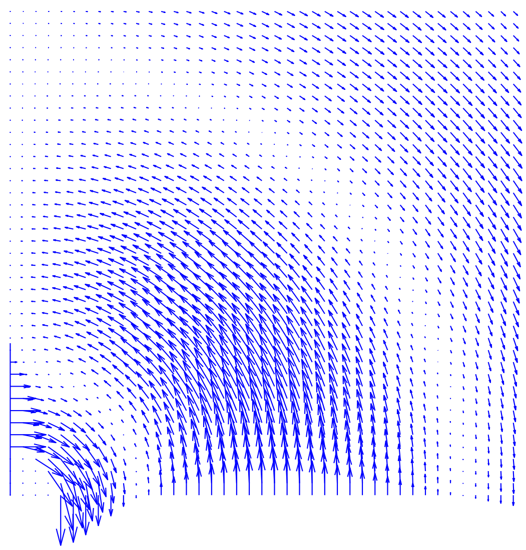

Figure 2: Orientation of the electric field in the case of the second picture of figure 1. The magnetic field is orthogonal to the page. To facilitate the view, a portion near the origin has been omitted since the intensity there is to strong. The signal is always zero when it reaches the north pole, and will stay zero in all the successive free evolution. The Poynting vector is always directed outbound.

The arms of the antenna are used as guides to delimitate the area. Note however that they are not “wave-guides” as it is usually intended, because the information on these conductors, being related to a standing wave, does not shift. Such a distinction is not emphasized in the literature, where antenna devices are often treated as a prolongation of the wave-guide carrying the original signal (see for instance [14], chapter XI). This simplifies a lot the analysis especially in numerical computations, but does not explain the mechanism of the antenna, based essentially on its resonant properties with consequent local production of standing waves. What we are examining here is exactly the possibility to combine both effects.

Due to properties that we will try to better clarify later in section 4, an electric field tends to “stick” at the boundary surface. This adherence is very pronounced, at the level that it is very difficult for a conductor to get rid of the signal. This should explain why the transition area has to be a resonant cavity with a fictitious spherical boundary supplied through the antenna device.

Indeed, let us note that the second term on the right-hand side of (2.19) vanishes for $r = d/2$, i.e. all the points of the corresponding spherical surface are of nodal type. The electric normal component adheres to the antenna wires, but fades when approaching the endpoints attenuating the strength of the link (see figure 2). As the endpoints are reached ($r = d/2$) the boundary field is zero. The electromagnetic wave has no more contact with the emitting device and, since it has been prepared to support fields with nonzero divergence, can finally fly away.

Note that the Poynting vector is always oriented radially; the energy flow moves from the central point at the speed of light and does not change trajectory. This conclusion is different from that obtainable theoretically by examining divergenceless fields (see for instance [8]), where the radiation patterns in the near-field tend to approach the arms of the antenna. Nevertheless, the whole procedure might not be as simple as we are claiming, so that more remarks will be added in the coming sections.

3 Coaxial antennas

We now work in cylindrical coordinates (r, ϕ, z). It turns out that a family of solutions of the whole set of model equations is:

$$\mathbf{E} \ = \ \Big(cf(r)g(t - z/c), \ 0, \ 0\Big)$$

$$\mathbf{B} \ = \ \Big(0, \ f(r)g(t - z/c), \ 0\Big) \qquad\qquad \mathbf{V} \ = \ (0, \ 0, \ c) \tag{3.20}$$

where f and g can be arbitrary. As the orientation of $\mathbf{V}$ testifies, the wave shifts along the z-axis solving equation (1.13). In general, the electric field is non divergenceless and we can only obtain $\rho = 0$ if $f(r) = 1/r$. Inside a coaxial cable we can handle the situation $\rho = 0$ by assuming suitable boundary conditions. In this case we are supposing that there is no dielectric between the internal wire and the external shield. The lines of force of the magnetic field are coaxial circumferences.

As the signal reaches the end of the cable one should expect an ejection like that of water from a garden hose. It is well-known however that this does not happen. Why? Because we are disregarding the two principles stated in the previous section. First of all, if we want the wave to get out and survive it is necessary to create a transition area in order to ensure the condition $\rho \neq 0$. In fact, as the effect of the boundary constraints ceases, the electric field has to damp transversally to zero and this cannot be realized if ρ remains equal to zero. The second issue concerns the contact with the boundary. The strength of the link must diminish until it vanishes. The wave must be accompanied to the exit through the creation of a resonant cavity. Part of the cavity boundary must be fictitious, representing the open way out of the ejected wave.

In order to better quantify the behavior at the end of a coaxial cable, a further extension of the model equations is needed (see [3] and [4]). Equation

(1.4) is improved as follows:

$$\rho\left(\frac{D\mathbf{V}}{Dt} \ + \ \mu(\mathbf{E}+\mathbf{V}\times\mathbf{B})\right) \ = \ -\nabla p \tag{3.21}$$

which is the non-viscous Euler equation for the velocity field $\mathbf{V}$. Among the unknowns we have now a new entry, i.e. the potential p, that, up to multiplicative dimensional constant, can be regarded as a pressure (the same radiation pressure measured when light hits an obstacle). The substantial derivative $D\mathbf{V}/Dt$ is defined as $\partial\mathbf{V}/\partial t+(\mathbf{V}\cdot\nabla)\mathbf{V}$. We also have a coupling constant μ, dimensionally equivalent to charge/mass, which has been estimated to be approximately equal to 2.3×10^{11} Coulomb/Kg. Note the strong analogy between (1.5), (3.21) and the equations ruling plasma physics (see [7], p.491). The novelty here is that there are no particles involved, but everything is related to the evolution of pure fields. The density of particles is now substituted by ρ.

Equation (3.21) amounts to an abstract generalization of the Lorentz law. When the field balance $\mathbf{E}+\mathbf{V}\times\mathbf{B}=0$ is not realized, there are accelerations imparted to the flow-field $\mathbf{V}$, which may change both in magnitude and direction. The corresponding wave does not freely propagate and the energy follows new velocity patterns. Condition $\mathbf{E}\cdot\mathbf{V}$ (see (1.6)) is not fulfilled, as it happens for instance during the propagation in a medium different from vacuum. In this case the electric field has a component in the direction of motion. We have free-waves when $D\mathbf{V}/Dt=0$ and $p=0$; in this case, if no external factors are acting on the wave, the signal propagates along straight rays.

Let us try to understand what happens as a signal arrives at the extreme of a coaxial cable. At the very end, a discontinuity tends to be created in the outgoing transversal electric field, due to the sudden disappearance of the boundary. This has the effect of breaking the balance $\mathbf{E}+\mathbf{V}\times\mathbf{B}=0$, creating a region where $\rho\neq 0$. Thanks to (3.21) there is a deviation of the velocity stream-lines with a corresponding generation of pressure. It can be shown (using the same arguments followed in [3], p.117, also confirmed by the numerical tests in [5]) that the divergence of $\mathbf{V}$ (initially zero in the coax) becomes positive inducing a spread of the signal. The electric field rotates and sticks on the outer conducting shield (note that the magnetic lines of force remain circumferences). This forms a kind of bottleneck. Part of the energy is rejected and returns back to the source. Therefore, no significant electromagnetic emission is observed. Estimates of the radiated power, obtained by using classical arguments, are given in [12].

Thus, an antenna is needed to drive the signal in the proper way (similar arguments are developed in [11], but always within the context of fields with zero divergence). Let us also observe that, by virtue of equations (3.21), a finer analysis can be made on the dipole. There, we assumed that the stream-lines determined by $\mathbf{V}$ were straight and originating from the small gap between the wires, but this interpretation is probably too simplistic. Due to the presence of a dielectric inside the feeding cable, condition (2.15) is not realized. This means that in the near-field the transformation of the signal has to take care of the

impedance adaptation. When the correct ratio of the intensities of the electric and magnetic fields is not respected, one has $\mathbf{E} + \mathbf{V} \times \mathbf{B} \neq 0$. As we said, this implies that an acceleration is imparted on the velocity field modifying a bit the trajectories. Through a feedback process, an adjustment of the electromagnetic field is made via (1.1) and (1.2) until the right impedance is reached. The behavior of photons in proximity of matter has been investigated in [10], section 3.6, where estimates based on truncated expansions point out the difficulties emerging when one imposes the condition $\rho = 0$ in the vacuum regions around matter.

One also has to deal with the fact that the arms are not perfect conductors and in their neighborhood the speed of propagation of the signal is less than c. Moreover, because of the asymmetry of the term $\mathbf{E} + \mathbf{V} \times \mathbf{B}$, the acceleration vector may change orientation according to the sign of the electric field, which varies in time, recalling the sprouts of an oscillating sprinkler. In other terms, $\mathbf{V}$ is not going to be a stationary velocity field. Although the far-field tends asymptotically to be distributed on spherical surfaces, the energy can slightly oscillate transversally with the frequency of the emitted wave. In order to recover more information, numerical simulations can be carried out by solving the full set of equations involving the simultaneous computation of $\mathbf{E}$, $\mathbf{B}$, $\mathbf{V}$ and p. In this way, we are not solving a wave equation, but a set of equations that couple electromagnetism with non-viscous fluid dynamics, where the velocity field indicates the direction of the energy flow. These are: (1.1), (1.2), (1.3) and (3.21), forming a closed system with a total of 10 relations and 10 unknowns.

Going back to the coaxial case, let us see if we can devise a system to actually expel the signal. The simplest idea is to use a device as shown in figure 3, where the hot wire is connected to a vertical wire (a quarter wave-length long, i.e.: $d/2 = \pi c/2\omega$) and the shield to a horizontal conductive disk. For a given frequency, the setting has to form a kind of resonant hemisphere in such a way that the intensity of the electric field, at the end of the wire and at the border of the disk, is zero. The electromagnetic signal coming out from a small gap between the wire and the disk is expected to be driven along conical pathways.

In order to compute the optimal size of the disk, one has to solve a wave equation on it. We assume that the electric field stays orthogonal to the disk and we denote by u the only component different from zero. We then eliminate from the Laplace operator in cylindrical coordinates the derivatives contributions with respect to z and ϕ. Of course, this is just an approximation since we still do not know the exact shape of the cavity. In truth, the resonant zone should be determined through a global process where also the vertical wire is taken into consideration. Nevertheless, we arrive at the equation:

$$\frac{\partial^2 u}{\partial t^2} = \frac{c^2}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial u}{\partial r}\right) \tag{3.22}$$

whose solutions are related to Bessel functions. Our suggestion is to take $u = \sin(\omega t) Y_0(\omega r/c)$, where Y_0 is a Bessel function of the second kind (presenting a singularity at the origin). The radius of the disk has to be determined in

correspondence of the first zero of Y_0, which is $0.893c/\omega$ (we recall the $2\pi c/\omega$ is the total wavelength). Hence, the ratio between the length of the wire and the radius of the disk is about 1.76. Of course, these conclusions are referred to the hypothetical case of a perfect device.

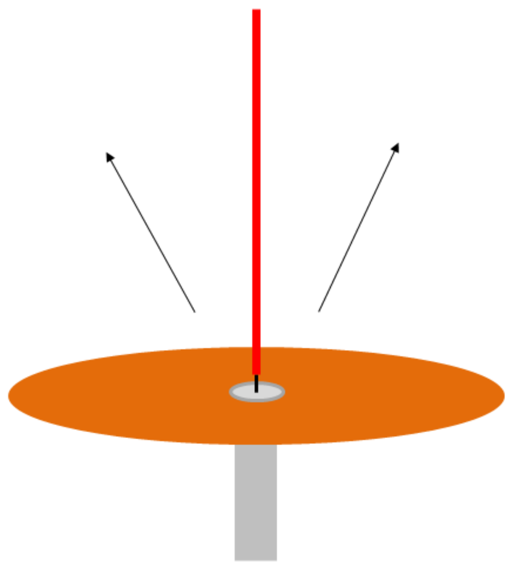

Figure 3: The output of a coaxial cable is connected to a plate and a vertical wire (monopole antenna). The ejected signal is expected to follow conical paths. The magnetic field is divergenceless and distributed on coaxial circumferences. For a given frequency, the optimal power emanated should be obtained when the whole system forms a resonant cavity, having both the boundary of the disk and the endpoint of the wire as nodal sets.

The above information may be used for dimensioning the ground of a monopole antenna (for an overview the reader is addressed for instance to [15]). Early experimental results, for many choices of the disk diameter, were conducted in [13]. From the theoretical viewpoint, the monopole is usually regarded as a half-space dipole. Reliable theoretical estimates are thus available (see also [9], p.4-28, in particular for what concerns the radiation patterns). The fact that the radiation is directed into the upper half tends to be justified by the reflecting properties of the ground. Our explanation is indeed quite different from the qualitative viewpoint. Take however into account that the new set of equations includes the space of solutions of the standard Maxwell's system, therefore all previous results and conclusions still hold true.

4 Further details and speculations

We recall once again that requiring the divergence of the electric field to be nonzero is not just a mathematical abstraction but a hypothesis one cannot avoid. If we mechanically move the plates of a charged parallel capacitor (no dielectric in between), the intensity of the field varies from a constant to another. The process does not occur instantaneously, since the information needs some time to propagate. Inevitably, there will be regions inside the capacitor where the electric field, whose lines of force are assumed to be orthogonal to the plates, is not constant and $\rho \neq 0$. It is true that ρ is going to be extremely small due to the fact that the information travels at the speed of light. But, when we deal with a coaxial cable operating at high frequency, a change of size, perhaps due to a joint, may have an appreciable influence on ρ.

The study of a capacitor under the influence of oscillating fields is clearly explained with plenty of details in the Feynman's book (see [2], vol.2, chapter 23). The antenna gap between the two arms of the dipole forms a tiny capacitor, which is exposed to an oscillating potential, whose frequency is not the resonant one but not too far from it. The presence of values of ρ different from zero is not negligible at this stage. A similar condition is presumably encountered in the sparks generated by an Hertzian coil, also present in the early versions of the Marconi's antenna. In [2], for a capacitor composed by two disks connected at a generator with a pair of wires, the analysis is carried out as a function of the distance from the symmetry axis. However, no indications about the longitudinal behavior are provided, so skipping the conclusions about ρ.

For an infinitely extended parallel capacitor, subject to a homogeneous difference of potential $V_0 \sin(\omega t)$ we can explicitly show some computations. The horizontal plates are located at $z = \pm a$. It is the matter of solving the wave equation $\partial^2 V/\partial t^2 = c^2 \partial^2 V/\partial z^2$, with the boundary conditions $V(t, \pm a) = \pm\frac{1}{2}V_0 \sin(\omega t)$. We then get the solution:

$$V(t, z) \;=\; \frac{V_0 \sin(\omega t)\sin(\omega z/c)}{2\sin(\omega a/c)} \tag{4.23}$$

that clearly also depends on z. By taking the gradient one recovers the third component of the electric field that, up to multiplicative constant, behaves as $E_3 = \sin(\omega t)\cos(\omega z/c)$. The divergence of this field is certainly nonzero. If the argument of the cosinus is small (plates at short distance or low-frequency) the electric field only changes with time, remaining practically constant in space. Instead, in the gap of a dipole antenna (which is in general not so small compared to the entire device) ρ starts assuming meaningful values.

By computing the curl of the field $\mathbf{E} = (0, 0, E_3)$, by (1.2) one recovers that the induced magnetic field $\mathbf{B}$ must satisfy $\partial\mathbf{B}/\partial t = 0$. The whole setting is incompatible with Maxwell's equations, even if both $\mathbf{E}$ and $\mathbf{B}$ solve independently a vector wave equation. Here the inconsistency is due to the assumption that the infinite plates are uniformly supplied. So, as done in [2], we should suppose that the plates are disks supplied at the center. Computations are now more

involved, since together with the dependence on the variable z we also have to allow $\mathbf{E}$ to be non-parallel. Having $\rho \neq 0$, we cannot rely on Maxwell's equation and we are obliged to use the new set of equations. The signal produced in this semi-resonant regime is neither fish nor fowl, being a combination of some fields in phase with other fields out of phase. The results turn out to be more complex, but at the same time more realistic (so we think). What happens just outside the small gap of the antenna is difficult to predict without numerical simulations. As we said, part of the signal flows along the arms and another part is ejected, after passing through a transition area. A correct design of the feeding gap is a necessary step to guarantee the optimality of the output; in fact this small component has a crucial role in the antenna performances (see for instance [1] or [9]).

In section 2, we mentioned about the "sticking" properties of the electric field on a metallic surface. Let us formalize better this concept. Consider for simplicity an electric field $\mathbf{E}$ orthogonal to a surface and not depending on time (for instance the constant field inside a parallel capacitor in the stationary regime). The field is zero inside the conductor, implying the existence of a discontinuity. For a more accurate analysis, one should enter the atomic structure and explain the reasons of this sharp variation. We stay however vague with this respect. We just assume that, within a distance comparable to a few Angstroms, the electric field drops from a constant to zero, producing a sharp boundary layer. Of course $\rho \neq 0$ within this neighborhood. Take now $\mathbf{B} = 0$ and $\mathbf{V} = 0$ in the model equations. The only relation surviving is (3.21), which yields: $\mu\rho\mathbf{E} = -\nabla p$. Using that $\mathbf{E}$ is orthogonal to the surface, one gets: $p = -\mu|\mathbf{E}|^2$. Therefore, a sort of negative "pressure" can be felt near the surface, also if there are no physical particles exerting it. We claim that this is responsible of the effective mechanical movement of the charged plates, commonly attributed to Coulomb attraction, explaining how such an action-at-a-distance may occur in practice. As far as the antenna is concerned, this reasoning should clarify why the signal remains glued to the device until $\mathbf{E} = 0$. These heuristic arguments are very unconventional but efficacious.

The last issue we would like to discuss here is what happens at the end of the antenna arms. We said that each endpoint is a node for the transversal electric field. This cannot be true for the magnetic field which is out of phase of 90 degrees. One may notice that this situation is in contrast with the absence of current at the endpoints. But, if one argues in terms of pure electromagnetic fields, thus excluding the existence of an actual flow of particle charges (such as electrons), there is no contradiction. We only have to explain what happens to the magnetic component as the wire ends. We recall that our wire has a diameter equal to 2ϵ. We guess that on top of the upper arm the nonzero oscillating magnetic field is coupled with a newborn electric field, both enclosed in a small bubble forming a thin cap of the same diameter of the wire. Both fields decay fast to zero with the the distance from the endpoint. Such isolated region does not interfere with the emitted wave. Again, these speculative considerations can be verified with appropriate numerical tests. One can also try to figure out

the behavior of the fields in the farthest sections, when the length of the dipole is not a multiple of half wave-length (see for instance [9], p.4-6).

5 Future projects

In this section we would like to fantasize about some ambitious projects. Let f in (3.20) be a continuous function vanishing outside a specified interval. Thus, the support of the corresponding wave is a cylinder. We are now dealing with a perfect monochromatic electromagnetic soliton. Can a signal of this type be produced in nature? Can we build a source capable of emitting such a wave? Do antennas with infinite directivity exist? If this was true, we would be able to communicate point-to-point without reaching other targets and with no energy dissipation.

According to the experience this seems not feasible. Maybe it is true: the question has no answer. Maybe it is only matter of exploiting new territories. Now we have a new set of model equations. Now we also have a new set of recipes teaching us how to design an antenna. The result must come from a symbiosis of laboratory tests and numerical computations. The agenda is open. The creation of very directive signals, certainly useful in applications, would also validate the theoretical model.

The suggestions given here are addressed to optimize the antenna configuration for a given emitted frequency. Of course we expect that the same apparatus may continue to work within a certain range of frequencies (bandwidth), though with some losses. Ordinary applications show us that antennas can do a reasonable job under conditions that are far from optimality. Nevertheless, there is always an inverse relation to be observed, connecting frequency and the reciprocal of the size of the emitting device. It is difficult to get around this restriction.

It seems impossible for instance to produce radio-waves on the range of one KHz with a tool a few centimeters wide. There are several reasons indicating that this is hard to achieve. We first need a source where oscillating electric and magnetic fields are produced with no phase lag. This cannot be a capacitor under the action of an alternate voltage. The effect would be very mild (see previous section). However, we can independently combine electric and magnetic fields, generated by individual sources, to get the desired effect. So, this problem can be somehow circumvented. Afterwards, there is the question of pre-regulating ρ in order to prepare the wave to the open space. As we said, a capacitor subject to an alternate field must produce internal regions with $\rho \neq 0$, although ρ is terrifically small at low frequencies. Interposed materials with variable dielectric constants may help to this purpose. Anyway, the most difficult task is to get rid of the signal and in order make it possible one cannot account on resonance phenomena. We know however what should be done: the electric signal has to reach the emission site with zero intensity on the edge of the device, in order to lose contact with it. Once discovered the ill,

somebody may come out with a medicine. Solving the problem of transmitting low-frequency radio-waves in small space is important to allow the transfer of energy at a distance and in large amount. The top is to realize this goal with the maximum directivity.

References

[1] Collin R., Zucker F., *Antenna Theory, Part I*, McGraw-Hill (1969).

[2] Feynman R.P., Leighton R., Sands M., *The Feynman Lectures on Physics*, Addison-Wesley (1964).

[3] Funaro D., *Electromagnetism and the Structure of Matter*, World Scientific, Singapore (2008).

[4] Funaro D., Electromagnetic radiations as a fluid flow, arXiv:0911.4848 v1.

[5] Funaro D., Numerical simulation of electromagnetic solitons and their interaction with matter, J. Sci. Comput., **45** (2010), pp.259-271.

[6] Funaro D., A Lagrangian for electromagnetic solitary waves in vacuum, arXiv:1008.2103 v1.

[7] Jackson J.D., *Classical Electrodynamics*, Second Edition, John Wiley & Sons (1975).

[8] Jackson J.D., How an antenna launches its input power into radiation: The pattern of the Poynting vector at and near an antenna, American J. of Phys., **74-4** (2006), pp.280-288.

[9] Johnson R.C., *Antenna Engineering Handbook*, Third Edition, McGraw-Hill (1993).

[10] Keller O., On the theory of spatial localization of photons, Physics Reports, **411** (2005), pp.1-232.

[11] McDonald K.T., Radiation in the near zone of a center-fed linear antenna, Preprint, Princeton University (2004).

[12] McDonald K.T., Radiation from the open end of a coaxial cable, Preprint, Princeton University (2009).

[13] Maley S.W., King R.J., Impedance of a monopole antenna with a circular conducting-disk ground system on the surface of a lossy half space, J. of Research of the Nat. Bureau of Standards, **65D-2** (1961), pp.183-188.

[14] Schelkunoff S.A., *Electromagnetic Waves*, D. van Nostrand Company, New York (1943).

[15] Weiner M.M., *Monopole Antennas*, CRC Press (2003).